Grand Strategy in
The Settlers of Catan

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Grand Strategy in "The Settlers of Catan" follows an inductive process to establish the best strategies in the game "The Settlers of Catan." This paper will go step-by-step using statistical and mathematical methods to model situations and establish the best strategies to maximize victory in the game.

Goal and Assumptions

First, the goal must be defined, and like the game Settlers of Catan, the goal of this paper is to find the best strategies for achieving 10 "Victory Points" (how one wins the game) in the shortest amount of time. But because each player starts out with 2 settlements, each player begins with 2 VPs, and consequently only needs to earn 8 VPs.

The game of Catan can also be played in many variations, all of which have a significant impact on the strategies which can be employed, and the effectiveness of varying strategies. Changing the number of players, for example, can affect the length of a game, productivity per turn, and the necessary steps for victory. Likewise, adding an expansion pack can introduce new pieces, resources, and rules, all of which can have a dramatic effect on how one earns necessary VPs.

For the purposes of this paper, a standard game of four players will be used. There will be no expansion packs employed, and all standard rules will be followed.

Methods of Earning Victory Points

Once again, the goal is to find a strategy which allows one to acquire 8 VPs in as few turns as possible (since players begin with 2, and a total of 10 is necessary to win). As such, the ways by which an agent (player) earns VPs must be defined. These ways are as follows:

1. **Building a Settlement**: This action, which costs 1 lumber, 1 brick, 1 wheat, and 1 sheep. The payoff for this action is 1 VP. $S = l + b + w + s$; where a settlement, $S = 1$ VP.
2. **Building a City**: This action which costs 2 wheat and 3 ore (defined as variable r to eliminate confusion with zero). As such: $C = 2w + 3r$; where a city, $C = 1$ VP. This definition may seem odd, given that in Catan, a city is worth 2 victory points. The true value of these 2 wheat and 3 ore, however, is only 1 VP, since the combination of these resources only adds 1 VP to an agent's total VP count.
3. **Victory Point Development Card**: There are 25 Development Cards (DPs), which are shuffled randomly at the beginning of

the game. Of the 25 DPs, 5 are Immediate Victory Points, and the others create victory points in varying combinations. For the 5 IVPs: IVP $= l + w + s$; where 1 IVP = 1 VP.

4. **Largest Army**: The Largest Army Card occurs when an agent acquires 3 knight cards, and is worth 2 VPs.
5. **Longest Road**: The Longest Road Card occurs when an agent acquires the longest road over 5 consecutive roads, worth 2 VPs.

Basic Value of Resources

A large matrix can be helpful when determining the value of each resource. To use a large matrix, a system of equations is necessary; four equations, in fact, are necessary to determine the value of the five resources, because lumber and brick are, functionally, the same resource. The value equations for the *Settlement, S,* and *City, C,* are fairly simple because there is no probability involved. The sum of the resources simply has a value equal to 1 VP.

The second most challenging equation to derive is for the Immediate Victory Point Cards, drawn from the Development Card deck. A simple expected value analysis, however, reveals the necessary values. $P(IVP) = \frac{5}{25} = 0.20$; this implies there is a 0.20 probability of receiving 1 VP on a random draw in the shuffled Development Card deck. As such, the equation $IVP = w + s + r$ can be used because paying 1 wheat, 1 sheep, and 1 ore, has an expected value of 0.20 VP.

The fourth most difficult equation to derive is the **Largest Army** card. This card has a value of 2 VPs, and is acquired by drawing and playing 3 Knight cards from the deck. This equation, while not used in the matrix, is helpful to derive now for future analysis.

In a standard Development Card deck for Catan, there are 25 cards, 14 of which are Knight cards. There is not, however, replacement, which makes the drawing of cards not independent, and the probability of drawing a Knight card always changing. For purposes of simplicity in determining the value of a Knight card in terms of resources, only one agent draws from the deck, and replacement occurs. This allows for a Binomial setting, in which the binary options are Knight or Non-Knight, the draws are independent because of replacement, there are a set number of trials (at n=3, since this is a reasonable number of development cards purchased by a player per game), and the set number of successes is 3.*

$$P(\textit{Largest Army in 3 draws} = 3w + 3s + 3r) =$$
$$= P(3\ \textit{Consecutive Knights}) = (\tfrac{14}{25})^3 = 0.176$$

*Although this binomial distribution does not account for the replacement of cards, it will be useful for later analyzing the value of strategies which place greater or lesser emphasis on the purchasing of Development Cards, and consequently varying emphasis on the Largest Army card, the Longest Road card, and Immediate Victory Point cards.

An expected value analysis can be completed, in which there is a 0.176 probability of drawing three knights to acquire the Largest Army card. As such,

$$2 * 0.176 = 3w + 3s + 3r$$
$$0.352 = 3w + 3s + 3r$$

Finally the most difficult equation to predict the value of is the Longest Road card, because its value can vary based on individual agent preferences and the preferences and strategies of other agents. For one player in a typical game of Catan, the vast majority of their roads are used. As such, an approximation of 13 out of the 15 roads will be used. Another necessary assumption is that players will build roads consecutively. This, however, is a reasonable assumption given the added benefit of a long consecutive road. Another essential note is that players begin with 2 roads, so they must construct 11 roads to meet the postulated 13 necessary to obtain the longest road card at the conclusion of the game.

$$\textit{Longest Road} = 2\ vp = 11\ \textit{roads} = 11l + 11b$$

Now that our five equations have been established, matrices can be constructed to solve for the value of each resource. They are multiplied as shown below to find the approximate value of each resource. There arises an issue, however, because there is no equation which distinguishes lumber and brick, making the two goods of equivalent value. As a result, a 4 by 4 matrix should be used, initially combining brick, *b*, and lumber, *l*, because they are, functionally, the same resource.

$$[vp\ recipes]\ X\ [variables] = [vp\ values]$$

$$[A]\ X\ [B] = [C]$$

$$[A]^{-1}\ X\ [C] = [B]$$

Matrix [A]: [*vp recipes*]

$$\begin{bmatrix} 2 & 1 & 1 & 0 \\ 0 & 2 & 0 & 3 \\ 0 & 1 & 1 & 1 \\ 22 & 0 & 0 & 0 \end{bmatrix}$$

Matrix [B]: [*variables*]

$$\begin{bmatrix} l\ and\ b \\ w \\ s \\ r \end{bmatrix}$$

Matrix [C]: [*vp values for recipes*]

$$\begin{bmatrix} 1 \\ 1 \\ 0.2 \\ 2 \end{bmatrix}$$

Resultant Matrix:

$$l = 4.55$$

$$b = 4.55$$

$$w = 142.73$$

$$s = -60.91$$

$$r = -61.82$$

Values have been multiplied by 100 for simplicity in analysis.

Note the apparently peculiar nature of some of these values. Lumber and brick may seem reasonable in stark contrast to enormous value of wheat, and negative values of sheep and ore. However, the large value of wheat illustrates how valuable a resource it truly is. Wheat can be used to construct settlements, cities, and Development Cards, all of which produce VPs. So it is not so surprising that wheat has this value. The magnitude of the values of sheep and ore show how the resources are valuable in the production of VPs. The negative signs, however, serve to illustrate how they are worthless without other resources, name wheat and each other. Thus, while sheep and ore are technically the least valuable resources, the magnitudes of the values permit the conclusion that the resources are most essential in the follow order:

1. *Wheat*
2. *Ore*
3. *Sheep*
4. *And Brick and Lumber*

Integrating Turns

Now that the value of the resources has been determined, the typical number of turns must be established. A 90% confidence interval will be created for the length of a turn, in an attempt to capture the true mean of turn lengths .

The goal is to establish an interval which captures the true mean length in an efficiently played game of Catan, with a point estimate of 78 seconds based on a random sample of the lengths of turns in an efficiently played game of Catan. A one sample-t interval will be used because the goal is to generate a confidence interval for a mean. Because the sample is random, $n \leq 10\%$ of all turns taken, and the plot of the data reveals no skew or outliers, it is relatively safe to assume a normal distribution *N(78, 43)*. As such, there is a 90% confidence level that the interval from 49.1 to 107 captures the true mean of turn length in Catan. The broadness of this interval illustrates how turns in Catan can take a variety of times. But also that the point estimate, 78 seconds, is an approximately accurate estimate, or average, that can be used.

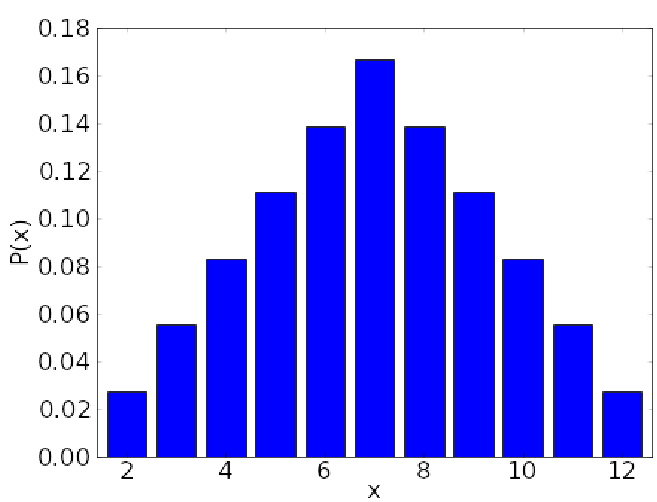
Based on empirical data, a game of catan typically lasts 90 minutes, as such:

$$90\ min\left(\frac{60\ seconds}{1\ min}\right)\left(\frac{1\ turn}{78\ seconds}\right)\left(\frac{1}{4\ players}\right) = 17.3\ turns$$

Essentially, players have approximately 17 turns to accumulate 10 victories if playing efficiently assuming a generally accepted game time of 90 minutes. An agent, however, must consider the trade off of playing in too many turns. An agent may decrease the number of turns it takes for him or her to accumulate 10 VPs, but he or she runs into many challenges in doing so, namely a scarcity of resources, coming off as too aggressive to other players, and hedging their bets on certain numbers on the board… the next component.

The Board

The board in the Settlers of Catan is a complex system. When considering the board, one must take into account resources, probabilities, and others' strategies (where it gets particularly interesting). The most basic element of the board are the die, and the probabilities associated with the resource hexes. As is commonly known, while not a normal distribution, when x is the number of die, $\lim_{x \to \infty} P(x)$ bears a stronger resemblance to a normal distribution. In the following chart, X is a discrete variable, occupying integer values from 2 to 12, with varying probabilities.



Distribution 1: Die Probabilities

Knowledge from the resource-value analysis earlier provides key insight into strategic choices of initial piece placement. The game begins when each player has placed two settlements, and two roads, in the following order: player A, B, C, D, D, C, B, A, in order to balance the quality of the resources one achieves. But several things must be taken into account when placing settlements: the resources around the settlements and the probabilities on those settlements. As a result, the initial value of a hex is determined by the following equation, where R is the is the value of the resource found in the resource-value analysis, h is the number of hexes which a single settlement touches, and P is the probability of the die landing on that resource (ie $P(6\ total\ on\ die) = 0.14$):

$$V_{single} = \sum_{1}^{h} R_n P_n$$

Note the $R(Desert) = 0$.

It is worth noting, however, that the total value of a players set-up can be determined by a similar equation, where H as opposed to h, represents the total hexes a player's settlements touch after set-up is complete:

$$V_{total} = \sum_{1}^{H} R_n P_n$$

Thus, it is always favorable to emerge from the set-up process with $V_{total} > 0$, which implies that the player's set up would typically generate the resources necessary for VP production, and that expanding on a player's set-up can only increase a player's capacity for production.

Below are six randomly generated scenarios, which illustrate the value of randomly placed settlements using a simulation. The surrounding hexes are denoted with their resource variable, and the corresponding numbers on those hexes are in the right column, the probabilities below. The bottom boxed value is the V_{single} for each of the scenarios.

Simulation 1

s,s,l	**9,6,11**
-60.91	0.1111
-60.91	0.1389
4.55	0.0556
	-14.97

l,b,w	**10,8,12**
4.55	0.0833
4.55	0.1389
142.73	0.0278
	4.98

r,r,l	**5,8,3**
-61.82	0.1111
-61.82	0.1389
4.55	0.0556
	-15.20

s,w,D	**4,8,0**
-60.91	0.0833
142.73	0.1389
0	0
	14.75

s,r,D	**11,8,0**
-60.91	0.0556
-61.82	0.1389
0	0
	-11.97

b,b,l	**12,6,5**
4.55	0.0278
4.55	0.1389
4.55	0.1111
	1.26

Rationalizing the Counterintuitive

For experienced players, these values may seem obsolete. A seasoned Catan player may argue that all resources are of equal value, as they all contribute to VPs in some way. These values, however, represent the value of each resource with respect to VPs. An experienced player would laugh at the thought of wheat having a value of 142, while sheep and ore are relegated to 60. In a rational world, however, these values make sense.

Without other resources, namely wheat, ore and sheep are worthless. There is no single use of ore *or* wheat which does not require wheat in some fashion. As such, it is logical that wheat is so valuable, and that only wheat, when combined with sheep and ore, extract value.

One may then argue, "why not place one's settlements on all wheat then, from a purely rational standpoint." Several reasons. (1) It is not practical. No maker of a board of Catan will actually place many wheats close together. Most players would be hard pressed to place their initial settlements on 3 or more wheat hexes. (2) The rational player would also realize that this is a foolish strategy. While wheat is the most valuable resource, a player would be remiss to believe that he or she could win a game without a variety of resources.

The fact remains, however, that wheat makes the hex. Only the hexes *Simulation 1* with wheat had a positive value in the end, with the exception of the brick, brick, and lumber hex, which had a small positive value of 1.26.

And when it comes to brick and lumber, the only reason that on their own they have positive values, is because they are essentially the same resource. There is not a single item which requires *only* brick or *only* lumber, but together they can make a road.

Theorem #1: Wheat Value Theorem

Any hex with wheat with a reasonable probability of selection automatically makes that hex more valuable than the surrounding hexes.

Axiom 1: Sheep and ore are worthless without wheat.

Theorem #2: Brick n' Lumber Theorem

Brick and lumber are the same resource, given that there are no items which require only brick or only lumber.

Theorem #3: Set-Up Value Theorem

The value of a player's set up is defined as:

$$V_{total} = \sum_{1}^{H} R_n P_n$$

Axiom 1: The value of a single *settlement*, at any time, can be defined:

$$V_{single} = \sum_{1}^{h} R_n P_n$$

A Chi-Square test illustrates that the value of a player's initial set up and the outcome of the player's game are not independent. As such, the value of a player's initial set up is correlated with the probability of that player winning.

χ^2 Test for independence between $V_{total} = \sum_{1}^{H} R_n P_n$ and the number of times that player wins and loses:

H_o: There is no association between the initial value of a player's starting set-up and the number of times that player wins and loses.
H_a: There is an association between the initial value of a player's starting set-up and the number of times that player wins and loses.

$\alpha = 0.05$ as a standard alpha-level.

Conditions:

1. Random: The player was randomly selected among all players at the Catan World Championships (CWC), and the games analyzed were randomly selected among the games that player played at the CWC.
2. Large Counts: While none of the expected values exceed five, they are all relatively close, so, while there are some trepidations, it is still relatively safe to carry on the Chi-Square test for independence. (see values below)

3. Independent: The games analyzed were less than 10% of the games played by this player at the CWC.

$$\chi_c^2 = \sum \frac{(O_i - E_i)^2}{E_i}$$

Data

	$V \le -1($	$-10 \le V$	$V \le 10$	Total
# of Wins	1	3	7	11
# of Losses	8	4	1	13
Total	9	7	8	**24**

Expected Values

	$V \le -10$	$-10 \le V \le 10$	$V \le 10$
# of Wins	4.1	3.2	3.7
# of Losses	4.9	3.8	4.3

$\chi^2 = 9.99$; $p = 0.0068$

Conclusion: Because the alpha-level is greater than the p-value, we have convincing evidence that value of a player's initial set-up and the number of wins and losses are not independent. As such, a higher initial set-up value tends to be associated with a greater percentage of wins.

Long-Run Strategy

The following equation models a player's total VP at any given point in the game:

$$\text{VP} = S + 2C + 2LA + 2LR + IVP$$

Where, S is the number of settlements, C is the number of cities, LA is the number of largest army cards, LR is the number of long road cards, and IVP is the number of immediate victory point cards a player has at any given time. As such, the goal is get VP = 10.

So the task becomes to maximize the VP equation in as few resources as possible. As such, each of the components can be broken down into their resources. It is worth noting that the values shown below are multiplied by 100, so the number of VP which this equation is trying to reach is 1,000:

$$\text{VP} = S + 2C + 2LA + 2LR + IVP$$

$$\text{VP} = (l + w + b + s) + 2(l + 3w + b + s + 3r) + 2(3w + 3s + 3r)/0.176 + (11l + 11b) + (w + s + r)/0.2$$

$$\text{VP} = 14l + 46.1w + 14b + 42.1s + 45.1r$$

$$1000 = 14l + 46.1w + 14b + 42.1s + 45.1r$$

Theorem #4: Resource Value Theorem
The total value of a player's hand and pieces is equal to:

$$\text{VP} = 14l + 46.1w + 14b + 42.1s + 45.1r$$

* *Values have been multiplied by 100 for simplicity of analysis to avoid decimals and arithmetic errors.*

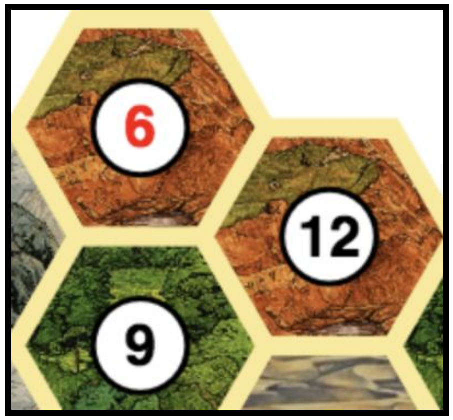
Strategy by Strategy Analysis

Each strategy type: Road, City, Development Card, and Balanced, has its merits and pitfalls. Some lead to overdependence on a single resource, while others constrain growth. References will be made to the following randomly generated board to provide insight into each specific strategy.

Board A

Type I Strategy: Longest Road Strategy

This strategy places great emphasis on the building of roads, and consequently the acquiring of lumber and brick. Players typically place their settlements in locations with brick and lumber. The following area found at the top of board A would, for example, attract a player comfortable with the Longest Road Strategy.



Application of **Axiom 1** of the Set-Up Value Theorem reveals that a settlement placed on this hex has a $V_{single} = 1.26$. So, while the player may home resources from this particular settlement with a probability of 0.2778, the resources which the player takes home ultimately contribute very little towards the ultimate goal of winning by acquiring a total of 10 VPs.

A Type I player of this nature would likely select a secondary settlement location with resources that diversify that player's holdings, like wheat, sheep, and ore.

For a player of this type, the VP equation can begin to take form, since it is highly unlikely a Type I player would acquire the resources necessary for the Largest Army or IVP, since there is no overlap in these kinds of players' primary resources of consumption:

$$\text{VP} = S + 2C + 2LA + 2LR + IVP$$

$$\text{VP} = S + 2C + 2LR$$

$$\text{VP} = (l + b + w + s) + 2(l + 3w + b + s + 3r) + 2(11b + 11l)$$

$$\text{VP} = 25l + 25b + 6w + 3s + 6r$$

As such, a Type I player must compliment the Longest Road card with cities and settlements.

Theorem #5: Road Builder Theorem

A player with a preference for building roads, settlements, and acquiring the Longest Road card gains VPs via the following equation, where the goal is to have VP = 1000.

$$\text{VP} = 25l + 25b + 6w + 3s + 6r$$

As can be seen, the Type I player values lumber and brick far more than other types of players, since they are essential to the player's strategy.

Type II Strategy: City Building Strategy

This strategy places greater emphasis on the construction of cities, which are 2 VPs. Because a city requires $C = 2w + 3r$ it can be inferred that wheat and ore are the most essential resources for this strategy. Since, as established by the Chi-Square Test earlier, there is a correlation between the starting value of a player's set-up and the number of wins and losses. Below is a 2 sample z test which illustrates the difference between the proportion of all available cities built by all players and the proportion of all available cities built by players employing the City Building Strategy.

2 Sample z Test for Number of Difference in the Proportion of Available Cities Constructed

Parameter: $p_c - p_n$ is the true difference between the two proportions, where p_c is the proportion of available cities built by City Strategy Players and p_n is the proportion of available cities built by non-City Strategy Players.

Statistic: $\widehat{p}_c - \widehat{p}_n = 0.90 - 0.75 = 0.25$

$H_o: p_c - p_n = 0$

$H_a: p_c - p_n \geq 0$

$\alpha = 0.10$; as a standard alpha-level.

Procedure: 2 Sample z Test for Number of Difference in the Proportion of Available Cities Constructed

Conditions:

1. Random: The players whose proportions of cities built for both the City Building Strategy and the non-City Building strategy samples were randomly selected.
2. Independence: The players selected for both the City Building Strategy sample and the non-City Building strategy sample are both greater than one-tenth of the entire number of players of the respective strategy at the CWC.
3. Large Counts: While none of the values are greater than 10, the purpose of the large counts is to determine if the size of the sample is appropriate relative to the size of the population. While this condition is not explicitly met, subjective analysis reveals that the sample size $n_c = 10$ and $n_n = 10$ is an appropriate size relative to the population of players at the CWC.

a. Average $p_c = 3.8/4 = 0.90(10) = 9.0;\ 0.10(10) = 1$

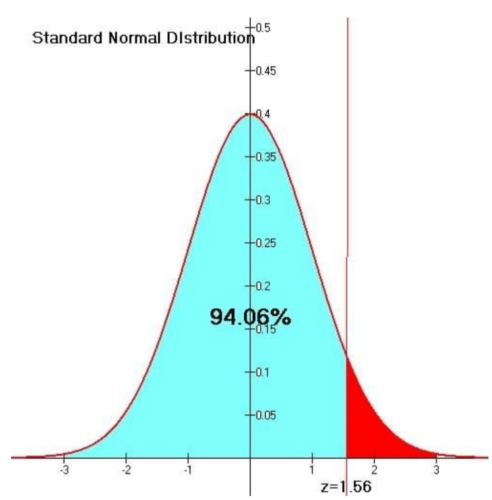
b. Average $p_n = 3.0/4 = 0.75(10) = 7.5;\ 0.25(10) = 2.5$

Mean: $\mu_{\hat{p}_c - \hat{p}_n} = 0$

*Must use pooled proportion

Stat: $Z = (Stat - null)/SD =$ $Z = \dfrac{(\hat{p}_1 - \hat{p}_2) - 0}{\sqrt{\hat{p}(1-\hat{p})\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$ $= 1.566$

$p = 0.0594$



Conclusion: Since our p-value is less than the alpha level, we have convincing evidence that there is a difference in the true proportion of available cities built by players using the City Building Strategy, and players not using the city building strategy. The City Building Strategy typically leads to approximately 0.25 more cities built.

This is significant because when rounded to the nearest whole city, the average number of cities built by City Building Strategy Players typically leads to approximately one more city than non-City Building Strategy Players. This is highly advantageous because the difference of one city is equal to a difference of 2 VPs over the course of an entire game, which can be a significant margin when the game comes to a finish, as an experienced player would know.

As a result, this Type II strategy is best for players who aspire to have a high initial value set-up and are comfortable consolidating their resource production to focus on wheat and ore. It is statistically advantageous because, as demonstrated in the Chi-Square Test, high value initial set ups typically are associated with a greater number of wins. Because the Type II strategy requires players to set-up around wheat, which is the highest value resource, they will have a greater chance of winning.

Type III Strategy: Development Card Strategy

This final analysis does not focus on a single strategy as much as it is an auxiliary choice to purchase more development cards and earn the largest army card. Recall from before the Binomial setting, in which the binary options are Knight or Non-Knight, the draws are independent because of replacement, there are a set number of trials (at n=3, since this is a reasonable number of development cards purchased by a player per game), and the set number of successes is 3.*

$$P(\textit{Largest Army in 3 draws} = 3w + 3s + 3r) =$$
$$= P(3\ \textit{Consecutive Knights}) = \left(\tfrac{14}{25}\right)^3 = 0.176$$

*Although this binomial distribution does not account for the replacement of cards, it will be useful for later analyzing the value of strategies which place greater or lesser emphasis on the purchasing of Development Cards, and consequently varying emphasis on the Largest Army card, the Longest Road card, and Immediate Victory Point cards.

As the number of trials increase, the pattern below is observed:

# of Trials	Binomial Probability
3	0.176
4	0.309
5	0.340

The observed pattern, however, is even more significant because the probability of drawing an immediate victory point also increases. Suppose three knights have been drawn already, with replacement. The probability of drawing an IVP also increases.

# of Knights Drawn	*P*(IVP)
1	0.21
2	0.22
3	0.23

This is significant because it changes the expected value of any one development card, and increases the expected value as more cards are drawn, regardless of whether the cards drawn are knights or Immediate Victory Points.

Conclusion: The development card strategy is rewarding because of the victory points it has the capability of producing, but this is only at probability, while it may produce more than expected, it is equally as likely to produce less

VPs than expected. Thus, this Type III strategy is best for players willing to take a chance and hedge their victory on a bit of luck.

Overall Conclusion:

The value of wheat cannot be ignored, and because the Type II City Building Strategy makes the most use of wheat, this is the dominant strategy among the three strategies layed out in this paper. It is worth noting, that while I have taken into account a great deal of variables and thought through numerous situations of combinations and permutations of actions, there is still much more that can lead a player to victory, and more strategies to be explored.

But the concrete conclusions that cannot be ignored are the values of the resources. It clearly places the Wheat, Sheep, and Ore combination among the most valuable because this combination produces the greatest set-up value which can actually be built. Likewise, the conclusions with respect to the Immediate Victory Point Cards illustrate the gamble development cards pose, and how a conservative player uses development cards to produce the large army card, while a more liberal player uses the development cards for immediate victory points, despite the greater amount of risk.

Perhaps, you have learned a thing or two about the game of Catan, as it is my hope that through mathematical analysis, we can uncover non-trivial truths about the things we take for granted.

Appendix

1: Value Matrices

	Lumber and Brick	Wheat	Sheep	Ore
settlement	2	1	1	0
city	0	2	0	3
IVP	0	1	1	1
Largest Road	22	0	0	0
	0	0	0	045454545
	1.5	0.5	-1.5	.13636363
	-0.5	-0.5	1.5	045454545
	-1	0	1	090909090

l and b	1		2	1	1	0
w	1		0	2	0	3
s	0.2		0	1	1	1
r	2		22	0	0	0

0.0909	0.0455
1.4273	0.0455
-0.6091	1.4273
-0.6182	-0.6091
	-0.6182

l	4.55
b	4.55
w	142.73
s	-60.91
r	-61.82

2: Variable values

l	4.55	0.08	0.36
b	4.55	0.10	0.45
w	142.73	0.12	17.13
s	-60.91	0.14	-8.53
r	-61.82	0.12	-7.42
w	142.73	0.10	14.27
			16.27

w	142.73	0.03	4.28
w	142.73	0.03	4.28
w	142.73	0.03	4.28
w	142.73	0.03	4.28
w	142.73	0.03	4.28
w	142.73	0.03	4.28
			25.69

3: Time per Term Sample

14:50	15:38	48
18:05	18:36	31
26:24:00	27:32:00	68
42:31:00	43:30:00	60
48:20:00	49:25:00	65
57:37:00	59:37:00	120
5:51	8:27	156

Mean	SD
78.28571429	43.87373658

4: Die Combination Probability and random sample of hex set-ups

Value	# of Combos	robability
2	1	0.0278
3	2	0.0556
4	3	0.0833
5	4	0.1111
6	5	0.1389
7	6	0.1667
8	5	0.1389
9	4	0.1111
10	3	0.0833
11	2	0.0556
12	1	0.0278

Resource	Value
l	4.55
b	4.55
w	142.73
s	-60.91
r	-61.82

l,b,b	**9,6,12**
4.55	0.1111
4.55	0.1389
4.55	0.0278
	1.26

0.2778

s,s,l	**9,6,11**
-60.91	0.1111
-60.91	0.1389
4.55	0.0556
	-14.97

s,w,D	**4,8,0**
-60.91	0.0833
142.73	0.1389
0	0
	14.75

(4 con't)

Percent Chances
2.78
5.56
8.33
11.11
13.89
16.67
13.89
11.11
8.33
5.56
2.78

l,b,w	**10,8,12**
4.55	0.0833
4.55	0.1389
142.73	0.0278
	4.98

r,r,l	**5,8,3**
-61.82	0.1111
-61.82	0.1389
4.55	0.0556
	-15.20

s,r,D	**11,8,0**
-60.91	0.0556
-61.82	0.1389
0	0
	-11.97

b,b,l	**12,6,5**
4.55	0.0278
4.55	0.1389
4.55	0.1111
	1.26